

Practice Quiz (solutions)

1. Suppose a certain geometric figure has the cycle index polynomial

$$P_G(x_1, x_2, x_3, x_4) = \frac{1}{4}(x_1^4 + 2x_2 + x_2^2).$$

Suppose also, with 3 colors, the pattern inventory

$$P_G(r + w + b, r^2 + w^2 + b^2, r^3 + w^3 + b^3, r^4 + w^4 + b^4)$$

is equal to

$$\begin{aligned} r^4 + w^4 + b^4 + r^3w + r^3b + w^3r + w^3b + b^3r + b^3w + 2r^2w^2 + 2r^2b^2 + 2w^2b^2 \\ + 3r^2wb + 3w^2rb + 3b^2rw. \end{aligned}$$

- (a) How many distinguishable colorings use all three colors?
- (b) How many distinguishable colorings use exactly 2 colors?
- (c) How many distinguishable colorings don't use white?

Ans: (a) Adding the coefficients of the last 3 terms: 9 distinguishable colorings.

Ans: (b) Adding the coefficients of the 4th through 12th terms:

12 distinguishable colorings.

Ans: (c) Adding the coefficients of the terms $r^4 + b^4 + r^3b + b^3r + 2r^2b^2$:

6 distinguishable colorings.