Math 3103 Combinatorics (Luecking)   NAME:_____
                                                      (Please print clearly)
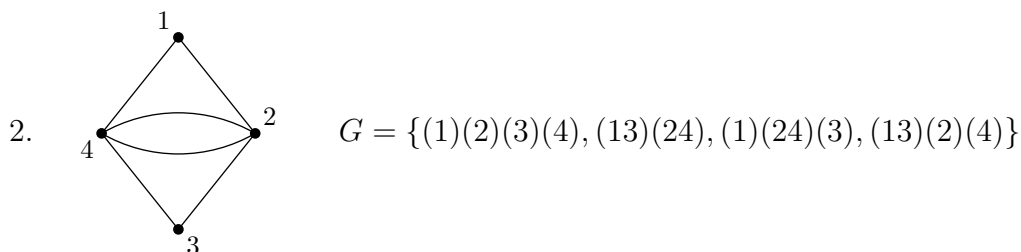Fourteenth Quiz (solutions)   Due April 24, 2024

Note: For the 2 figures below I have given the group of rigid motions, expressed as permutations of the vertex labels. Determine the number of distinguishable colorings of the vertice (a) when there are 3 colors to choose from, and (b) when there are 5 colors to choose from. Do not simplify either of these.

1.

$\{(1)(2)(3)(4)(5)(6), (14)(25)(36), (13)(2)(46)(5), (16)(25)(34)\}$
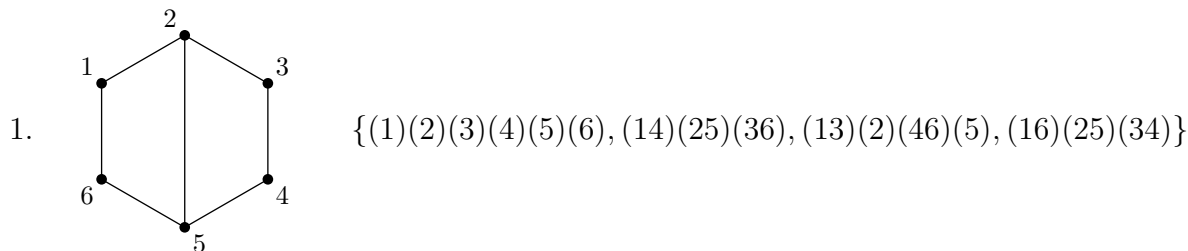
Ans: (a) $\frac{1}{4}(3^6 + 3^3 + 3^4 + 3^3)$, (b) $\frac{1}{4}(5^6 + 5^3 + 5^4 + 5^3)$

2.

$G = \{(1)(2)(3)(4), (13)(24), (1)(24)(3), (13)(2)(4)\}$

Ans: (a) $\frac{1}{4}(3^4 + 3^2 + 3^3 + 3^3)$, (b) $\frac{1}{4}(5^4 + 5^2 + 5^3 + 5^3)$

3. The following group of permutations represents the allowed rigid motions of a certain figure with 5 vertices.

   $G = \{(1)(2)(3)(4)(5), (1234)(5), (13)(24)(5), (1432)(5)\}.$

   (a) Write out the cycle index polynomial for G.

Ans: $\frac{1}{4}\left(x_1^5 + 2x_1 x_4 + x_1 x_2^2\right)$

   (b) How many distinguishable ways are there to color the vertices if 2 colors are available? Do not simplify.

Ans: $\frac{1}{4}\left(2^5 + 2 \cdot 2^2 + 2^3\right)$