Math 3103 Combinatorics (Luecking) NAME:______________________________
(Please print clearly)

Twelfth Quiz (solutions) Due April 15, 2024

Note: Unless otherwise indicated, you need not show any work.

1. Both of the following are *group* codes. For each one: **(i)** determine the minimum nonzero weight of the code words then, using this value, determine **(ii)** the maximum number of errors that can reliably be detected and **(iii)** the maximum number of errors that can reliably be corrected.

   (a) $\mathscr{C}_2 = \{(000\,000\,0000),\ (100\,100\,1011),\ (010\,010\,1101),\ (001\,001\,1110),$
   $(110\,110\,0110),\ (101\,101\,0101),\ (011\,011\,0011),\ (111\,111\,1000)\}$ in $\mathbb{Z}_2^{10}$

Ans: (i) The nonzero weights are 5, 6 and 7. The minimum is 5. This is the minimum distance so: (ii) 4 errors can always be detected. (iii) 2 errors can always be corrected.

   (b) $\mathscr{C}_1 = \{(000\,0000),\ (100\,1011),\ (010\,0111),\ (001\,1101),$
   $(110\,1100),\ (101\,0110),\ (011\,1010),\ (111\,0001)\}$ in $\mathbb{Z}_2^{7}$

Ans: (i) The minimum nonzero weight is 4. This is the minimum distance so: (ii) 3 errors can always be detected. (iii) 1 error can always be corrected.

2. The following is a generator matrix for a group code: $G = \left(\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right)$

   Do the following.

   (a) Find the code words corresponding to the messages $v = (1100)$ and $w = (1001)$.
   Ans: $vG = (1100011)$ and $wG = (1001101)$

   (b) Write out the parity check matrix H corresponding to G.

Ans: $H = \left(\begin{array}{cccc|ccc} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right)$

   (c) For the received words (i) $r = (1000\,001)$ and (ii) $s = (1000\,100)$ find Hr^{tr} and Hs^{tr}, then use those results to correct both if there is an error. If you decide this can't be done, give a reason.

Ans: $Hr^{\mathrm{tr}} = (111)^{\mathrm{tr}}$, the 3rd column of H, so change the 3rd bit of r to get $(1010\,001)$
$Hs^{\mathrm{tr}} = (010)^{\mathrm{tr}}$, the 6th column of H, so change the 6th bit of s to get $(1000\,110)$.

   (d) If possible, decode the corrected words in part (c) to obtain their original messages.

Ans: For r: (1010), and for s: (1000).