Math 3103 Combinatorics (Luecking) NAME:______________________
(Please print clearly)

Eleventh Quiz (solutions) Due April 5, 2024

Note: Please put all your answers on this sheet. Limit the work you show to what will fit. For the six 'how many' questions, your ultimate answer **must** be simplified to an explicit number.

1. Let $G = \mathbb{Z}_{30}$, with the operation of addition mod 30. Answer the following:
   (a) How many elements does G contain? **Ans:** 30

   (b) List all the elements in the cyclic subgroup $H = \langle 12 \rangle$. These must all be explicit elements of G. **Ans:** $\{12, 24, 6, 18, 0\}$

   (c) How many cosets of H are there in G? **Ans:** $|G|/|H| = 30/5 = 6$

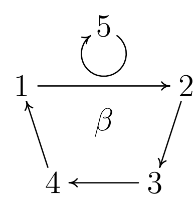
2. Let $G = u(\mathbb{Z}_{36})$, the group of units of the ring $\mathbb{Z}_{36}$ with the operation of multiplication mod 36. Answer the following:
   (a) How many elements does G contain? **Ans:** $\phi(36) = 36(1/2)(2/3) = 12$

   (b) List all the elements in the cyclic subgroup $H = \langle 13 \rangle$. These must all be explicit elements of G. **Ans:** $\{13, 25, 1\}$

   (c) How many cosets of H are there in G? **Ans:** $12/3 = 4$

3. Let $G = \mathcal{S}_5$, the group of all permutations of the set $\{1, 2, 3, 4, 5\}$. The operation is composition of permutations. Answer the following:
   (a) How many elements does G contain? **Ans:** $|G| = 5! = 120$

   (b) $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 1 & 5 \end{pmatrix}$ is an element of G. List **all** the elements in the cyclic subgroup $H = \langle \beta \rangle$. Express all the elements in the same notation I've used for β. (The figure might aid in visualizing β.)



Ans: $\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 1 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 1 & 2 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 1 & 2 & 3 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} \right\}$

[In cycle notation: $\{(1234)(5), (13)(24)(5), (1432)(5), (1)(2)(3)(4)(5)\}$.]

   (c) How many cosets of H are there in G? **Ans:** $|G|/|H| = 5!/4 = 30$