Math 3103 Combinatorics (Luecking) NAME:______________________________
(Please print clearly)

Tenth Quiz (solutions) Due March 25, 2024

1. Answer the following about $\mathbb{Z}_{2024}$. These answers must be completely simplified.

   (a) Given the prime factorization $2024 = 2^3 \cdot 11 \cdot 23$, how many units does this ring have?

Ans: $\phi(2024) = 2^3 \cdot 11 \cdot 23 \left(\frac{1}{2}\right)\left(\frac{10}{11}\right)\left(\frac{22}{23}\right) = 2^2 \cdot 10 \cdot 22 = 880$

   (b) How many proper zero divisors?

Ans: $2024 - \phi(2024) - 1 = 2024 - 880 - 1 = 1143$

   (c) Using the Euclidean algorithm, find the inverse of 101 in the ring $\mathbb{Z}_{2024}$. This answer must be an explicit element of $\mathbb{Z}_{2024}$.

Ans: The Euclidean algorithm gives $\begin{cases} 2024 = 20(101) + 4 \\ 101 = 25(4) + 1 \end{cases}$ or $\begin{cases} n = 20k + r_1 \\ k = 25r_1 + r_2 \end{cases}$
where $n = 2024$, $k = 101$, $r_1 = 4$ and $r_2 = 1$.

Eliminate r_1 and solve for r_2 to get $r_2 = 501k - 25n$ or $1 = 501(101) - 25(2024)$.

This says $1 = 501 \cdot 101$ in $\mathbb{Z}_{2024}$ and so $101^{-1} = 501$.