Math 3103 Combinatorics (Luecking) NAME:______________________
(Please print clearly)

Ninth Quiz (solutions) Due March 13, 2024

1. For the following recurrence relation problem, find the generating function for the sequence a_n without actually finding a_n.

$$a_n - 4a_{n-1} + 2a_{n-2} = (2)3^n\,,\quad n \geq 2\,.$$
$$a_0 = 2,\quad a_1 = 5.$$

Ans: From the recurrence relation we get

$$\sum_{n=2}^{\infty} a_n x^n - 4\sum_{n=2}^{\infty} a_{n-1}x^n + 2\sum_{n=2}^{\infty} a_{n-2}x^n = 2\sum_{n=2}^{\infty}(3x)^n$$

or $F(x) - 2 - 5x - 4x(F(x) - 2) + 2x^2F(x) = \dfrac{2(3x)^2}{1-3x}$.

whence $F(x) - 4xF(x) + 2x^2F(x) = 2 - 3x + \dfrac{2(3x)^2}{1-3x}$.

and so $F(x) = \dfrac{2 - 3x + \dfrac{2(3x)^2}{1-3x}}{1 - 4x + 2x^2}$.

2. Do the following computations in the ring $\mathbb{Z}_{19}$. All answers must be elements of $\mathbb{Z}_{19}$, completely simplified. Note: the operations $+$ and $\cdot$ are those defined on $\mathbb{Z}_{19}$, **not** the usual operations on integers.

(a) $10 + 10 + 4$ (b) $5 \cdot 4 \cdot 10$ (c) $(7 + 15) \cdot 8$

(d) -5 (e) 5^{-1} (trial and error is OK)

Ans: (a) $(10 + 10) + 4 = 1 + 4 = 5$

(b) $(5 \cdot 4) \cdot 10 = 1 \cdot 10 = 10$

(c) $(7 + 15) \cdot 8 = 3 \cdot 8 = 5$

(d) $-5 = 14$ (because $5 + 14 = 0$)

(e) $5^{-1} = 4$, because $5 \cdot 4 = 1$.