Math 3103 Combinatorics (Luecking) NAME:________________________
(Please print clearly)

Eighth Quiz (solutions) Due March 8, 2024

For each of the following nonhomogeneous recurrence relations with initial conditions, do the following.

(a) Find the homogeneous solution.

(b) Find a particular solution.

(c) Find the solution that satisfies the initial conditions.

1. $a_n - 4a_{n-1} + 4a_{n-2} = (4/9)3^n$, $n \geq 2$,
$a_0 = 0$, and $a_1 = 0$.

Ans: (a) From $r^2 - 4r + 4 = 0$ we get a double root $r = 2$. So, $a_n^{(h)} = C_1 2^n + C_2 n 2^n$.

(b) Setting $a_n = A3^n$ we get $A3^n - 4A3^{n-1} + 4A3^{n-2} = (4/9)3^n$, from which $A/9 = 4/9$ and $A = 4$. So, $a_n^{(p)} = (4)3^n$.

(c) From $a_n = C_1 2^n + C_2 n 2^n + (4)3^n$ we get $C_1 + 4 = 0$ and $2C_1 + 2C_2 + 12 = 0$ giving $C_1 = -4$ and $C_2 = -2$. So, $a_n = -(4)2^n - 2n2^n + (4)3^n$.

2. $a_n - 5a_{n-1} + 4a_{n-2} = 6$, $n \geq 2$,
$a_0 = 0$, and $a_1 = 0$.

Ans: (a) From $r^2 - 5r + 4 = 0$ we get roots 1 and 4. So, $a_n^{(h)} = C_1 + C_2 4^n$.

(b) Setting $a_n = An$ we get $An - 5A(n-1) + 4A(n-2) = 6$, from which $-3A = 6$ and $A = -2$. So, $a_n^{(p)} = -2n$.

(c) From $a_n = C_1 + C_2 4^n - 2n$ we get $C_1 + C_2 = 0$ and $C_1 + 4C_2 - 2 = 0$ giving $C_1 = -2/3$ and $C_2 = 2/3$. So, $a_n = -2/3 + (2/3)4^n - 2n$.