Math 3103 Combinatorics (Luecking) NAME:_____
(Please print clearly)

Sixth Quiz (solutions) Due February 28, 2024

1. For each of the following first-order recurrence relations, find the solution that satisfies the given initial condition.

   (a) $a_n = a_{n-1} + 6\,,\ \ n \ge 1\,,$ **Ans:** $a_n = 5 + 6n\,.$
   $a_0 = 5\,.$

   (b) $a_n = 7a_{n-1}\,,\ \ n \ge 1\,,$ **Ans:** $a_n = 8(7^n)\,.$
   $a_0 = 8\,.$

   (c) $a_n = (3n+4)a_{n-1}\,,\ \ n \ge 1\,,$
   $a_0 = 2\,.$

   Ans: $a_n = (3(1)+4)\,(3(2)+4)\,(3(3)+4)\cdots(3n+4)\,2\,.$ Or $a_n = 2\prod_{j=1}^{n}(3j+4)\,.$

   (d) $a_n = a_{n-1} + 9n^2\,,\ \ n \ge 1\,,$
   $a_0 = 1\,.$

   Ans: $a_n = 1 + 9(1^2) + 9(2^2) + 9(3^2) + \cdots + 9n^2\,.$ Or $a_n = 1 + \sum_{j=1}^{n} 9j^2\,.$

2. For the following second-order recurrence relation,

   $a_n - a_{n-1} - 2a_{n-2} = 0\,,\ \ n \ge 2\,,$
   $a_0 = 2\,,\ \ a_1 = 7\,,$

   (a) write out the characteristic equation, **Ans:** $r^2 - r - 2 = 0.$

   (b) find the roots of the characteristic equation, **Ans:** $r = -1$ and $r = 2.$

   (c) and write the general solution of the recurrence relation. **Ans:** $a_n = C_1(-1)^n + C_2 2^n$