

You do not have to simplify numerical answers, but you must present your final answer using only numbers and the operations of addition, subtraction, multiplication, division, powers and factorials.

All solutions must appear on this sheet, in the spaces provided below each problem.

1. The 10-character string "STATUESQUE" contains 'S' 2 times, 'T' 2 times, 'U' 2 times and 'E' 2 times. The remaining 2 characters occur 1 time each.

- (a) How many 10-character strings are arrangements of this string?

Ans: $N = \frac{10!}{2!2!2!2!}$

- (b) How many of the strings in part (a) contain *none* of the substrings "SS", "TT", "UU", "EE"?

Ans: Using conditions

$$c_1 = \text{'contains "SS"'} , c_2 = \text{'contains "TT"'} , c_3 = \text{'contains "UU"'} , c_4 = \text{'contains "EE"'}$$

we get

$$S_1 = 4N(c_1) = 4 \frac{9!}{2!2!2!}, \quad S_2 = 6N(c_1c_2) = 6 \frac{8!}{2!2!},$$

$$S_3 = 4N(c_1c_2c_3) = 4 \frac{7!}{2!}, \quad S_4 = N(c_1c_2c_3c_4) = 6!.$$

So,

$$N(\bar{c}_1\bar{c}_2\bar{c}_3\bar{c}_4) = N - S_1 + S_2 - S_3 + S_4 = \frac{10!}{2!2!2!2!} - 4 \frac{9!}{2!2!2!} + 6 \frac{8!}{2!2!} - 4 \frac{7!}{2!} + 6!.$$

- (c) How many of the strings in part (a) contain *at least one* of the four substrings in part (b)?

Ans: $S_1 - S_2 + S_3 - S_4 = 4 \frac{9!}{2!2!2!} - 6 \frac{8!}{2!2!} + 4 \frac{7!}{2!} - 6!$

- (d) How many of the strings in part (a) contain *exactly one* of the four substrings in part (b)?

Ans: $S_1 - 2S_2 + 3S_3 - 4S_4 = 4 \frac{9!}{2!2!2!} - 2 \cdot 6 \frac{8!}{2!2!} + 3 \cdot 4 \frac{7!}{2!} - 4 \cdot 6!$