

**Instructions:** You need not simplify, but you must write your answer using only numbers and the operations of addition, subtraction, multiplication, division, power and factorial. For example, “ $5!/2!$ ” is OK, but “ $P(5,3)$ ” is not finished.

Write your answers on this sheet in the spaces provided.

1. A box labeled ‘ $A$ ’ contains 7 prizes, and a box labeled ‘ $B$ ’ contains 11 prizes. All the prizes are different.

(a) Suppose a contest winner is told to select **one** prize, to be chosen either from box  $A$  **or** from box  $B$ . How many possible outcomes are there?

**Ans:** There are 2 tasks: select from box  $A$  (7 possibilities), or select from box  $B$  (11 possibilities). Since the sets are disjoint there are  $7 + 11 = 18$  possibilities.

(b) Suppose the instructions are to select **two** prizes, one of them from box  $A$  **and** one of them from box  $B$ . How many possible outcomes are there?

**Ans:** These are the same 2 tasks, but *both* must be done. So, multiply the numbers  $7 \cdot 11 = 77$ .

(c) Suppose the instructions are to select **either** two prizes from box  $A$  **or** two prizes from box  $B$  (but not both). How many possible outcomes are there?

**Ans:** There are  $C(7,2)$  ways to pick from box  $A$  and  $C(11,2)$  ways to pick from box  $B$ . By the rule of sum there are:

$$C(7,2) + C(11,2) = \frac{7 \cdot 6}{2 \cdot 1} + \frac{11 \cdot 10}{2 \cdot 1} = 76 \text{ possible outcomes}$$

2. The 10-letter string "STATISTICS" has 3 occurrences of 'S', 3 occurrences 'T' and 2 occurrences of 'I', and no other repetitions.

(a) How many different 10-letter strings are arrangements of this string?

**Ans:**  $\frac{10}{3!3!2!}$ .

(b) How many arrangements of this string contain all three of the substrings "SSS", "TTT" and "II"?

**Ans:** Permutations of the 5 objects, 'SSS', 'TTT', 'II', 'A', and 'C':  $5!$  arrangements.