Math 3083 Linear Algebra (Luecking) NAME:________________________________
(Please print clearly)

Thirteenth Quiz (solutions) Due April 22, 2023

1. For the matrix $A = \begin{pmatrix} 4 & 0 & -3 \\ 0 & 3 & 0 \\ 2 & 0 & -1 \end{pmatrix}$, which has eigenvalues $\lambda = 1$, 2 and 3, find matrices S and D such that S is invertible and D is diagonal and $D = S^{-1}AS$. (Note: you are not required to find S^{-1}, although that would be one way to check your answers.)

Ans: For $\lambda = 1$, the matrix $A - 1I = \begin{pmatrix} 3 & 0 & -3 \\ 0 & 2 & 0 \\ 2 & 0 & -2 \end{pmatrix}$ reduces to $\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. So, the eigenspace consists of $\mathbf{x} = \begin{pmatrix} \alpha \\ 0 \\ \alpha \end{pmatrix}$, and a basis is $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$.

For $\lambda = 2$, the matrix $A - 2I = \begin{pmatrix} 2 & 0 & -3 \\ 0 & 1 & 0 \\ 2 & 0 & -3 \end{pmatrix}$ reduces to $\begin{pmatrix} 1 & 0 & -3/2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. So, the eigenspace consists of all $\mathbf{x} = \begin{pmatrix} 3\alpha/2 \\ 0 \\ \alpha \end{pmatrix}$ and a basis is $\begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix}$.

For $\lambda = 3$, the matrix $A - 3I = \begin{pmatrix} 1 & 0 & -3 \\ 0 & 0 & 0 \\ 2 & 0 & -4 \end{pmatrix}$ reduces to $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$. So, the eigenspace consists of all $\mathbf{x} = \begin{pmatrix} 0 \\ \alpha \\ 0 \end{pmatrix}$ and a basis is $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$.

Therefore, $S = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 1 & 2 & 0 \end{pmatrix}$ and $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$.