Math 3083 Linear Algebra (Luecking) NAME:______________________________

(Please print clearly)

Eleventh Quiz (solutions) Due April 8, 2024

1. Let the vector space $\mathbb{R}^4$ be given the inner product defined by
$\langle \mathbf{x}, \mathbf{y} \rangle = x_1y_1 + 2x_2y_2 + x_3y_3 + 2x_4y_4$.
Let $\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ -2 \\ 0 \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} -2 \\ 3 \\ 1 \\ 1 \end{pmatrix}$. Using this inner product, find the values of $\langle \mathbf{x}, \mathbf{y} \rangle$, $\|\mathbf{x}\|$, $\|\mathbf{y}\|$, and $\|\mathbf{x} + \mathbf{y}\|$.

Ans: $\langle \mathbf{x}, \mathbf{y} \rangle = (1)(-2) + 2(1)(3) + (-2)(1) + 2(0)(3) = 2$,
$\|\mathbf{x}\| = \sqrt{\langle \mathbf{x}, \mathbf{x} \rangle} = \sqrt{(1)^2 + 2(1)^2 + (-2)^2 + 2(0)^2} = \sqrt{7}$,
$\|\mathbf{y}\| = \sqrt{\langle \mathbf{y}, \mathbf{y} \rangle} = \sqrt{(-2)^2 + 2(3)^2 + (1)^2 + 2(1)^2} = \sqrt{25} = 5$, and
$\|\mathbf{x} + \mathbf{y}\| = \sqrt{\langle \mathbf{x} + \mathbf{y}, \mathbf{x} + \mathbf{y} \rangle} = \sqrt{(-1)^2 + 2(4)^2 + 2(-1)^2 + (1)^2} = \sqrt{36} = 6$.

2. Let $\mathbf{x} = \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$, $\mathbf{y} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ and $\mathbf{z} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$. Using the *standard scalar product* for the inner product on $\mathbb{R}^3$, find the projection of $\mathbf{z}$ onto the subspace $S = \operatorname{Span}(\mathbf{x}, \mathbf{y})$. Hint: $\mathbf{x} \perp \mathbf{y}$.

Ans: Since $\mathbf{x} \perp \mathbf{y}$, we can add the projections onto each of $\mathbf{x}$ and $\mathbf{y}$ separately:

$$\mathbf{p} = \frac{\mathbf{z}^T\mathbf{x}}{\mathbf{x}^T\mathbf{x}}\mathbf{x} + \frac{\mathbf{z}^T\mathbf{y}}{\mathbf{y}^T\mathbf{y}}\mathbf{y} = \frac{5}{9}\begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix} + \frac{-1}{9}\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} -11/9 \\ 7/9 \\ 8/9 \end{pmatrix}.$$