Math 3083 Linear Algebra (Luecking) NAME:______________________________
(Please print clearly)

Ninth Quiz (solutions) Due March 29, 2024

1. Let $\mathbf{u} = \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$.

(a) Find the scalar product of $\mathbf{u}$ and $\mathbf{v}$.

Ans: $\mathbf{u}^T\mathbf{v} = (3)(1) + (-1)(-2) + (4)(2) = 3 + 2 + 8 = 13$.

(b) Find $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$.

Ans: $\|\mathbf{u}\| = \sqrt{(3)^2 + (-1)^2 + (4)^2} = \sqrt{9 + 1 + 16} = \sqrt{26}$ and
$\|\mathbf{v}\| = \sqrt{(1)^2 + (-2)^2 + (2)^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$.

(c) If θ is the angle between $\mathbf{u}$ and $\mathbf{v}$, find $\cos\theta$. (Do *not* find θ itself.)

Ans: $\cos\theta = \dfrac{\mathbf{u}^T\mathbf{v}}{\|\mathbf{u}\| \cdot \|\mathbf{v}\|} = \dfrac{13}{3\sqrt{26}}$. Or $\dfrac{\sqrt{26}}{6}$. But not 0.8498.

(d) Find the vector projection of $\mathbf{u}$ onto $\mathbf{v}$.

Ans: $\mathbf{p} = \dfrac{\mathbf{u}^T\mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v} = \dfrac{13}{9}\mathbf{v} = \begin{pmatrix} 13/9 \\ -26/9 \\ 26/9 \end{pmatrix}$. Note: not $\begin{pmatrix} 1.44 \\ -2.89 \\ 2.89 \end{pmatrix}$.

(e) Find any nonzero vector in $\mathbb{R}^3$ that is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.

Ans: Such a vector must be a solution of the following system: $\begin{cases} x_1 - 2x_2 + 2x_3 = 0 \\ 3x_1 \ -1x_2 + 4x_3 = 0 \end{cases}$

This gives us, $\begin{aligned} x_1 &= (-6/5)x_3 \\ x_2 &= (2/5)x_3 \end{aligned}$

Now pick any nonzero value of x_3. For example, if $x_3 = 5$ then $\mathbf{x} = \begin{pmatrix} -6 \\ 2 \\ 5 \end{pmatrix}$.

An alternative is to compute $\mathbf{u} \times \mathbf{v} = \begin{pmatrix} 6 \\ -2 \\ -5 \end{pmatrix}$. Of course, these two are scalar multiples of each other. The cross product $\mathbf{u} \times \mathbf{v}$ makes sense only in $\mathbb{R}^3$ whereas solving a system can be done in any $\mathbb{R}^n$.