Math 3083 Linear Algebra (Luecking) NAME:______________________________
(Please print clearly)

Eighth Quiz (solutions) Due March 11, 2024

1. Let T be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$ given by $T(\mathbf{x}) = \begin{pmatrix} x_1 - 2x_2 + 3x_3 \\ 2x_1 - x_2 \end{pmatrix}$, where $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$. Find the matrix A that satisfies $A\mathbf{x} = T(\mathbf{x})$ for all $\mathbf{x}$ in $\mathbb{R}^3$.

Ans: The columns of A are the three vectors $T(\mathbf{e}_1) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $T(\mathbf{e}_2) = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$, *and* $T(\mathbf{e}_3) = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$, so $A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & -1 & 0 \end{pmatrix}$.

2. Let $\mathcal{E}$ be the standard ordered basis for $\mathbb{R}^3$ and let $\mathcal{C} = \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \end{pmatrix} \right]$, an ordered basis for $\mathbb{R}^2$. Let T be the linear transformation defined in problem 1. Find the representing matrix for T relative to $\mathcal{E}$ and $\mathcal{C}$.

Ans: The matrix obtained above is the representing matrix relative to the standard bases on both vector spaces. It therefore suffices to multiply UA, where U is the transition matrix from the standard basis of $\mathbb{R}^2$ to $\mathcal{C}$:
$U = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix}$.
That is, the representing matrix is UA or

$$\begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -2 & 3 \\ 2 & -1 & 0 \end{pmatrix} = \begin{pmatrix} -5 & 1 & 3 \\ 2 & -1 & 0 \end{pmatrix}$$