Math 3083 Linear Algebra (Luecking) NAME:______________________
(Please print clearly)

Seventh Quiz (solutions) Due March 8, 2024

1. The problems below are about the matrix $A = \begin{pmatrix} 1 & 1 & -2 & 0 & 2 \\ 2 & 1 & -3 & 1 & 2 \\ 1 & 0 & -1 & 1 & 0 \\ 1 & -2 & 1 & 0 & 2 \end{pmatrix}$. It has reduced echelon form $\begin{pmatrix} 1 & 0 & -1 & 0 & 2 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$. (Note: I have done the ERO calculations for you.)

   (a) Write down a basis for the row space of A:

Ans: Nonzero rows of the echelon form: $\{(1,0,-1,0,2),(0,1,-1,0,0),(0,0,0,1,-2)\}$

   (b) Write down a basis for the column space of A:

Ans: (b) Columns of A corresponding to leading variables: $\left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ -2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \right\}$

   (c) Find the rank and nullity of A (i.e., the dimensions of the column space and null space):

Ans: rank $= 3$, nullity $= 2$.

   (d) Find a basis for the nullspace of A:

Ans: The echelon form gives the following equations equivalent to $A\mathbf{x} = \mathbf{0}$:

$$\left.\begin{aligned} x_1 \quad - x_3 \quad + 2x_5 &= 0 \\ x_2 - x_3 \quad &= 0 \\ x_4 - 2x_5 &= 0 \end{aligned}\right\} \longrightarrow \left\{\begin{aligned} x_1 &= x_3 - 2x_5 \\ x_2 &= x_3 \\ x_4 &= \quad 2x_5 \end{aligned}\right.$$

Since x_3 and x_5 are free, putting $x_3 = 1$ and $x_5 = 0$ gives one basic solution; and putting $x_3 = 0$ and $x_5 = 1$ gives the other:

$$\begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} -2 \\ 0 \\ 0 \\ 2 \\ 1 \end{pmatrix}$$