Math 3083 Linear Algebra (Luecking) NAME:____________________________
(Please print clearly)

Fifth Quiz (solutions) Due February 28, 2024

1. For each of the following sets of vectors in $\mathbb{R}^3$ do the following: copy the vectors into a matrix and bring that matrix to echelon form using EROs. Then, based on that echelon form, answer these questions:
(i) Does the set span $\mathbb{R}^3$? **(ii)** Is the set independent? **(iii)** Is the set a basis for $\mathbb{R}^3$.
Note: All these questions must be answered "yes" or "no". Also, if done correctly, at most 4 EROs are needed for each.

(a) $\begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}1\\3\\5\end{pmatrix}$ **Ans:** $\begin{bmatrix}1&1\\2&3\\3&5\end{bmatrix} \xrightarrow[R_3-3R_1]{R_2-2R_1} \begin{bmatrix}1&1\\0&1\\0&2\end{bmatrix} \to \begin{bmatrix}1&1\\0&1\\0&0\end{bmatrix}.$

(i) No. (ii) Yes. (iii) No.

(b) $\begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}2\\1\\5\end{pmatrix}, \begin{pmatrix}3\\2\\7\end{pmatrix}$ **Ans:** $\begin{bmatrix}1&2&3\\0&1&2\\1&5&7\end{bmatrix} \to \begin{bmatrix}1&2&3\\0&1&2\\0&3&4\end{bmatrix}$

$\to \begin{bmatrix}1&2&3\\0&1&2\\0&0&-2\end{bmatrix} \to \begin{bmatrix}1&2&3\\0&1&2\\0&0&1\end{bmatrix}.$ (i) Yes. (ii) Yes. (iii) Yes.

(c) $\begin{pmatrix}2\\1\\0\end{pmatrix}, \begin{pmatrix}2\\3\\4\end{pmatrix}, \begin{pmatrix}4\\4\\4\end{pmatrix}$ **Ans:** $\begin{bmatrix}2&2&4\\1&3&4\\0&4&4\end{bmatrix} \to \begin{bmatrix}1&1&2\\1&3&4\\0&4&4\end{bmatrix} \to \begin{bmatrix}1&1&2\\0&2&2\\0&4&4\end{bmatrix}$

$\to \begin{bmatrix}1&1&2\\0&1&1\\0&4&4\end{bmatrix} \to \begin{bmatrix}1&1&2\\0&1&1\\0&0&0\end{bmatrix}.$ (i) No. (ii) No. (iii) No.

(d) $\begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\2\\2\end{pmatrix}, \begin{pmatrix}0\\-1\\-1\end{pmatrix}, \begin{pmatrix}0\\-1\\4\end{pmatrix}$ **Ans:** $\begin{bmatrix}1&1&0&0\\1&2&-1&-1\\1&2&-1&4\end{bmatrix}$

$\xrightarrow{\text{2 type III}} \begin{bmatrix}1&1&0&0\\0&1&-1&-1\\0&1&-1&4\end{bmatrix} \to \begin{bmatrix}1&1&0&1\\0&1&-1&0\\0&0&0&5\end{bmatrix} \to \begin{bmatrix}1&1&0&1\\0&1&-1&0\\0&0&0&1\end{bmatrix}.$

(i) Yes. (ii) No. (iii) No.