Math 3083 Linear Algebra (Luecking) NAME:__________________________
(Please print clearly)

Fourth Quiz (solutions) Due February 26, 2024

1. (a) Let $S_1 = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \middle| \; x_1 = x_2 + x_3, \text{ and } x_2 = -x_3 \right\}$. Find a matrix A such that $S_1 = \mathcal{N}(A)$

Ans: Several possibilities, the most straightforward being $A = \begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & 1 \end{pmatrix}$ based on S_1 being the set of solutions of the homogeneous system

$$x_1 - x_2 - x_3 = 0$$
$$x_2 + x_3 = 0$$

(b) Let $S_2 = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \middle| \; x_2 x_3 \le 0 \right\}$. Find two vectors in S_2 whose sum is not in S_2.

Ans: One possibility: $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$. Their sum is $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$.

(c) Let $S_4 = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \middle| \; x_1 + x_2 \ge x_3 \right\}$. Find a vector $\mathbf{v}$ in S_4 and a scalar α such that $\alpha\mathbf{v}$ is not in S_4.

Ans: Any nonzero vector in S_4 and any negative scalar. One possibility: $\mathbf{v} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $\alpha = -1$. The product is $-\mathbf{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$.

(d) Let $S_3 = \left\{ \begin{pmatrix} \alpha + 2\beta \\ -\alpha + \beta \\ 3\alpha + \beta \end{pmatrix} \middle| \; \alpha, \beta \text{ in } \mathbb{R} \right\}$. Find two vectors $\mathbf{a}$ and $\mathbf{b}$ in $\mathbb{R}^3$ such that $S_3 = \mathrm{Span}(\mathbf{a}, \mathbf{b})$.

Ans: One possibility: $\mathbf{a} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$, because $\alpha\mathbf{a} + \beta\mathbf{b} = \begin{pmatrix} \alpha + 2\beta \\ -\alpha + \beta \\ 3\alpha + \beta \end{pmatrix}$

2. For each of the example sets in problem 1, answer the following question: Is this set a subspace of $\mathbb{R}^3$?

Ans: (a) Yes, nullspace of a matrix. (b) No, not closed under addition. (c) No, not closed under scalar multiplication. (d) Yes, span of a set of vectors.