Math 3083 Linear Algebra (Luecking) NAME:________________________
(Please print clearly)

Third Quiz (solutions) Due February 7, 2024

1. Let A_1, A_2, and B be matrices and let A be the partitioned matrix $A = \left(\frac{A_1}{A_2}\right)$. Suppose

$$A_1B = \begin{pmatrix} 1 & 0 \end{pmatrix} \quad \text{and} \quad A_2B = \begin{pmatrix} 3 & 1 \\ -1 & 1 \\ 1 & 5 \end{pmatrix}.$$

(a) How many columns does B have? **Ans:** Same as A_1B and A_2B: 2 columns

(b) (i) How many rows does A_1 have? (ii) How many rows does A_2 have?

Ans: (i) Same as A_1B: 1 row. (ii) Same as A_2B: 3 rows.

(c) Find AB. **Ans:** $AB = \left(\frac{A_1B}{A_2B}\right) = \begin{pmatrix} 1 & 0 \\ 3 & 1 \\ -1 & 1 \\ 1 & 5 \end{pmatrix}$

2. Find the following determinants. Please produce completely simplified numbers.

(a) $\begin{vmatrix} 2 & 3 \\ -1 & 5 \end{vmatrix}$ (b) $\begin{vmatrix} 1 & 4 & 1 \\ 5 & 5 & 3 \\ 0 & 0 & 3 \end{vmatrix}$ (use row 3) (c) $\begin{vmatrix} 0 & 2 & 0 & 9 \\ 0 & 0 & 0 & 2 \\ 1 & 12 & 5 & 19 \\ 1 & 7 & 4 & 5 \end{vmatrix}$ (use row 2)

(d) $\begin{vmatrix} 32 & 11 & 7 & 21 \\ 1 & 1 & 2 & 3 \\ 2 & 2 & 4 & 6 \\ 41 & 7 & 10 & 5 \end{vmatrix}$ (compare rows) (e) $\begin{vmatrix} 1 & 2 & 3 & 3 \\ 1 & 4 & 4 & 5 \\ 2 & 4 & 7 & 11 \\ 3 & 6 & 9 & 5 \end{vmatrix}$ (use a few EROs).

Ans: (a) $(2)(5) - (-1)(3) = 13$

(b) Using the last row (or the partitioned matrix rule): $3 \cdot \begin{vmatrix} 1 & 4 \\ 5 & 5 \end{vmatrix} = 3 \cdot (-15) = -45$.

(c) Using cofactors of row 2, the determinant equals: $2(-1)^6 \begin{vmatrix} 0 & 2 & 0 \\ 1 & 12 & 5 \\ 1 & 7 & 4 \end{vmatrix}$.

Then, using row 1 of that 3×3, this equalss $(2)2(-1)^3 \begin{vmatrix} 1 & 5 \\ 1 & 4 \end{vmatrix} = -4(4-5) = 4$

(d) Third row is a multiple of the second: the determinant is 0.

(e) 3 type-III EROs turn this into: $\begin{vmatrix} 1 & 2 & 3 & 3 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & -4 \end{vmatrix} = 1 \cdot 2 \cdot 1 \cdot (-4) = -8$