Math 3083 Linear Algebra (Luecking) NAME:______________________
(Please print clearly)

Second Quiz (solutions) Due February 2, 2024

1. For the matrices $A = \begin{pmatrix} 1 & 0 & 2 \\ -1 & 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ -1 & 0 \\ 1 & 2 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ find the following *when possible*. If a combination is not possible, just write "not possible". (The matrix I_2 is the 2×2 identity matrix.)

   (a) $2C - AB$ **Ans:** $2C - AB = \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix} - \begin{pmatrix} 3 & 6 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} -1 & -2 \\ 6 & 6 \end{pmatrix}$.

   (b) CB **Ans:** not possible

   (c) $A^T + B$ **Ans:** $A^T + B = \begin{pmatrix} 1 & -1 \\ 0 & 1 \\ 2 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ -1 & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ -1 & 1 \\ 3 & 4 \end{pmatrix}$

   (d) $(C - I_2)A$ **Ans:** $(C - I_2)A = \begin{pmatrix} 0 & 2 \\ 3 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ -1 & 1 & 2 \end{pmatrix} = \begin{pmatrix} -2 & 2 & 4 \\ 0 & 3 & 12 \end{pmatrix}$

2. Find the inverse of the matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 4 \\ 3 & 1 & 4 \end{pmatrix}$ using elementary row operations.

   Note: the last line of your answer **must** be "$A^{-1} =$" followed by the appropriate 3×3 matrix.

Ans:
$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 2 & 1 & 4 & 0 & 1 & 0 \\ 3 & 1 & 4 & 0 & 0 & 1 \end{array}\right) \xrightarrow[R_3 - 3R_1]{R_2 - 2R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 1 & 1 & -3 & 0 & 1 \end{array}\right) \xrightarrow{R_3 - R_2}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 0 & -1 & -1 & -1 & 1 \end{array}\right) \xrightarrow{(-1)R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & -1 \end{array}\right) \xrightarrow[R_1 - R_3]{R_1 - 2R_3}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & -4 & -1 & 2 \\ 0 & 0 & 1 & 1 & 1 & -1 \end{array}\right), \text{ so } A^{-1} = \begin{pmatrix} 0 & -1 & 1 \\ -4 & -1 & 2 \\ 1 & 1 & -1 \end{pmatrix}$$