Math 3083 Linear Algebra (Luecking) NAME:__________________________
(Please print clearly)

First Quiz (solutions) Due January 29, 2024

1. Using only *elementary row operations* convert each of the following matrices to row-echelon form. Do this one ERO at a time. Note: each can be done with at most two EROs.

(a) $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 2 & -1 \\ 2 & 2 & 7 \end{pmatrix}$ **Ans:** $\xrightarrow{R_2 - R_1} \begin{pmatrix} 1 & 1 & 3 \\ 0 & 1 & -4 \\ 2 & 2 & 7 \end{pmatrix} \xrightarrow{R_3 - 2R_1} \begin{pmatrix} 1 & 1 & 3 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 3 \\ 0 & 2 & 6 \end{pmatrix}$ **Ans:** $\xrightarrow{R_3 - 2R_2} \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & 1 & -1 \\ 2 & 4 & 6 \\ 0 & 0 & 1 \end{pmatrix}$ **Ans:** $\xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 2 & 4 & 6 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{(1/2)R_1} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$

2. (a) Write out the *augmented matrix* for the following system of equations.

$$\begin{aligned} x_1 + x_2 + 2x_3 \phantom{- 3x_4} &= 1 \\ x_1 + x_2 - x_3 - 3x_4 &= 7 \\ -2x_1 - 2x_2 - x_3 + 3x_4 &= -8 \end{aligned}$$

**Ans:** $\left(\begin{array}{cccc|c} 1 & 1 & 2 & 0 & 1 \\ 1 & 1 & -1 & -3 & 7 \\ -2 & -2 & -1 & 3 & -8 \end{array}\right)$

(b) The reduced row-echelon form for that augmented matrix is $\left(\begin{array}{cccc|c} 1 & 1 & 0 & -2 & 5 \\ 0 & 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$.

Based on this reduced echelon form, (i) state which variables in the system are leading variables and (ii) state which ones are free variables.

**Ans:** (i) The leading 1's are in columns 1 and 3 so the leading variables are $x_1$ and $x_3$.
(ii) That leaves $x_2$ and $x_4$ as the free variables.

(c) Based on the matrix in (b), find all solutions of the system. Express your solution as a quadruple, where the free variable positions contain parameters and the leading variable positions are in terms of those parameters.

**Ans:** From part (b), $\begin{cases} x_1 + x_2 \phantom{+ x_3} - 2x_4 = 5 \\ \phantom{x_1 + x_2 +} x_3 + x_4 = -2 \end{cases}$ or $\begin{cases} x_1 = 5 - x_2 + 2x_4 \\ x_3 = -2 - x_4 \end{cases}$

Set the nonleading variables to arbitrary parameters: $x_2 = \alpha$ and $x_4 = \beta$. Then the solutions are: $(5 - \alpha + 2\beta, \alpha, -2 - \beta, \beta)$, where $\alpha$ and $\beta$ can be any numbers.