Math 3103 Combinatorics (Luecking)

Practice Quiz (solutions on page 2.)

1. The following group of permutations represents the allowed rigid motions of a certain figure with 5 vertices.

   $$G = \{(1)(2)(3)(4)(5), (123)(4)(5), (132)(4)(5), (12)(3)(45), (13)(2)(45), (1)(23)(45)\}.$$

   (a) Write out the cycle index polynomial for G.

   (b) How many distinguishable ways are there to color the vertices if 4 colors are available? Do not simplify.

2. Suppose a certain geometric figure has the cycle index polynomial

   $$P_G(x_1, x_2, x_3, x_4) = \frac{1}{4}(x_1^4 + 2x_4 + x_2^2).$$

   Suppose also, with 3 colors, the pattern inventory

   $$P_G(r + w + b, r^2 + w^2 + b^2, r^3 + w^3 + b^3, r^4 + w^4 + b^4)$$

   is equal to

   $$r^4 + w^4 + b^4 + r^3w + r^3b + w^3r + w^3b + b^3r + b^3w + 2r^2w^2 + 2r^2b^2 + 2w^2b^2 + 3r^2wb + 3w^2rb + 3b^2rw.$$

   (a) How many distinguishable colorings use all three colors?

   (b) How many distinguishable colorings use exactly 2 colors?

Ans: Problem 1.

(a) $P_G = \frac{1}{6}(x_1^5 + 2x_1^2 x_3 + 3x_1 x_2^2)$.

(b) $\frac{1}{6}(4^5 + 2 \cdot 4^3 + 3 \cdot 4^3)$.

Ans: Problem 2.

(a) Adding the coefficients of the last 3 terms: 9 distinguishable colorings.

(b) Adding the coefficients of the 4th through 12th terms:

12 distinguishable colorings.