Math 3103 Combinatorics (Luecking) NAME:________________________

(Please print clearly)

Tenth Quiz (solutions) Due November 10, 2023

Note: Please put all your answers on this sheet. Limit the work you show to what will fit. For the six 'how many' questions, your ultimate answer **must** be simplified to an explicit number.

1. Let $G = \mathbb{Z}_{40}$, with the operation of addition mod 40. Answer the following:

   (a) How many elements does G contain? **Ans:** 40

   (b) List all the elements in the cyclic subgroup $H = \langle 24 \rangle$. These must all be explicit elements of G. **Ans:** $\{24, 8, 32, 16, 0\}$

   (c) How many cosets of H are there in G? **Ans:** $|G|/|H| = 40/5 = 8$

2. Let $G = u(\mathbb{Z}_{40})$, the group of units of the ring $\mathbb{Z}_{40}$ with the operation of multiplication mod 40. Answer the following:

   (a) How many elements does G contain? **Ans:** $\phi(40) = 40(1/2)(4/5) = 16$

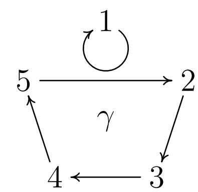
   (b) List all the elements in the cyclic subgroup $H = \langle 3 \rangle$. These must all be explicit elements of G. **Ans:** $\{3, 9, 27, 1\}$

   (c) How many cosets of H are there in G? **Ans:** $16/4 = 4$

3. Let $G = \mathcal{S}_5$, the group of all permutations of the set $\{1, 2, 3, 4, 5\}$. The operation is composition of permutations. Answer the following:

   (a) How many elements does G contain? **Ans:** $|G| = 5! = 120$

   (b) $\gamma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 4 & 5 & 2 \end{pmatrix}$ is an element of G. List **all** the elements in the cyclic subgroup $H = \langle \gamma \rangle$. Express all the elements in the same notation I've used for γ. (The figure might aid in visualizing γ.)



Ans: $\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 4 & 5 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 5 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 5 & 2 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} \right\}$

   (c) How many cosets of H are there in G? **Ans:** $|G|/|H| = 5!/4 = 30$