Math 3103 Combinatorics (Luecking) NAME:_________________________
(Please print clearly)

Ninth Quiz (solutions) Due October 25, 2023

1. Answer the following about $\mathbb{Z}_{2009}$. These answers must be completely simplified.

   (a) Given the prime factorization $2009 = 7^2 \cdot 41$, how many units does this ring have?

Ans: $\phi(2009) = 2009\left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{41}\right) = 7^2 \cdot 41 \left(\frac{6}{7}\right)\left(\frac{40}{41}\right) = 7 \cdot 6 \cdot 40 = 1680$

   (b) How many proper zero divisors?

Ans: $2009 - \phi(2009) - 1 = 2009 - 1680 - 1 = 328$

   (c) Using the Euclidean algorithm, find the inverse of 100 in the ring $\mathbb{Z}_{2009}$. This answer must be an explicit element of $\mathbb{Z}_{2009}$.

Ans: The Euclidean algorithm gives $\begin{cases} 2009 = 20(100) + 9 \\ 100 = 11(9) + 1 \end{cases}$ or $\begin{cases} n = 20k + r_1 \\ k = 11r_1 + r_2 \end{cases}$ where $n = 2009$, $k = 100$, $r_1 = 9$ and $r_2 = 1$ Eliminate r_1 and solve for r_2 to get $r_2 = 221k - 11n$ or $1 = 221(100) - 10(2009)$. This says $1 = 221 \cdot 100$ in $\mathbb{Z}_{2009}$ and so $100^{-1} = 221$.