

For each of the following nonhomogeneous recurrence relations with initial conditions, do the following.

- Find the homogeneous solution.
- Find a particular solution.
- Find the solution that satisfies the initial conditions.

1. $a_n - 4a_{n-1} + 4a_{n-2} = (2/9)3^n$, $n \geq 2$,
 $a_0 = 0$, and $a_1 = 0$.

Ans: (a) From $r^2 - 4r + 4 = 0$ we get a double root $r = 2$. So, $a_n^{(h)} = C_1 2^n + C_2 n 2^n$.

(b) Setting $a_n = A 3^n$ we get $A 3^n - 4A 3^{n-1} + 4A 3^{n-2} = (2/9)3^n$, from which $A/9 = 2/9$ and $A = 2$. So, $a_n^{(p)} = (2)3^n$.

(c) From $a_n = C_1 2^n + C_2 n 2^n + (2)3^n$ we get $C_1 + 2 = 0$ and $2C_1 + 2C_2 + 6 = 0$ giving $C_1 = -2$ and $C_2 = -1$. So, $a_n = -(2)2^n - n 2^n + (2)3^n$.

2. $a_n - 6a_{n-1} + 5a_{n-2} = 12$, $n \geq 2$,
 $a_0 = 0$, and $a_1 = 0$.

Ans: (a) From $r^2 - 6r + 5 = 0$ we get roots 1 and 5. So, $a_n^{(h)} = C_1 + C_2 5^n$.

(b) Setting $a_n = An$ we get $An - 6A(n-1) + 5A(n-2) = 12$, from which $-4A = 12$ and $A = -3$. So, $a_n^{(p)} = -3n$.

(c) From $a_n = C_1 + C_2 5^n - 3n$ we get $C_1 + C_2 = 0$ and $C_1 + 5C_2 - 3 = 0$ giving $C_1 = -3/4$ and $C_2 = 3/4$. So, $a_n = -3/4 + (3/4)5^n - 3n$.