Math 3103 Combinatorics (Luecking) NAME:______________________
(Please print clearly)

Sixth Quiz (solutions) October 9, 2023

1. Answer (a)–(c) below for the following recurrence relation problem.

$$a_n - 8a_{n-1} + 16a_{n-2} = 0 \quad (n \geq 2),$$
$$a_0 = 1\,, \quad a_1 = 7\,.$$

(a) Write down the characteristic equation and find its roots:

Ans: $r^2 - 8r + 16 = 0$ with double root $r = 4, 4$

(b) Write down a general solution:

Ans: $a_n = C_1 4^n + C_2 n 4^n$

(c) Find the solution that satisfies the initial conditions.

Ans: The initial conditions give $C_1 = 1$ and $4C_2 + 4C_2 = 7$ so that $C_2 = 3/4$. Thus

$$a_n = 4^n + \frac{3}{4} n 4^n$$

2. Answer (a)–(c) below for the following recurrence relation problem.

$$a_n - 2a_{n-1} + 2a_{n-2} = 0 \quad (n \geq 2),$$
$$a_0 = 2, \quad a_1 = 6\,.$$

(a) Write down the characteristic equation and find its roots:

Ans: $r^2 - 2r + 2 = 0$ with roots $r = 1 \pm i$

(b) Write down a general solution:

Ans: Either $a_n = C_1(1+i)^n + C_2(1-i)^n$ or

$$a_n = 2^{n/2}(C_1 \cos(45n) + C_2 \sin(45n))$$

(c) Find the solution that satisfies the initial conditions.

Ans: The initial conditions give $C_1 + C_2 = 2$ and $(1+i)C_2 + (1+i)C_2 = 6$ so that $C_1 - C_2 = 4/i$ giving $C_1 = 1 + 2/i$ and $C_2 = 1 - 2/i$. Thus

$$a_n = (1 + 2/i)(1+i)^n + (1 - 2/i)(1-i)^n$$

Altenatively, $a_n = 2^{n/2}(2\cos(45n) + 4\sin(45n))$