

Instructions: You need not simplify, but you must write your answer using only numbers and the operations of addition, subtraction, multiplication, division, power and factorial. That is, “ $5!/2!$ ” is OK, but “ $P(5, 3)$ ” is not finished.

1. A box labeled ‘ A ’ contains 8 toys, and a box labeled ‘ B ’ contains 12 toys. All the toys are different.

(a) Suppose a child is told to select **one** toy, to be chosen either from box A **or** from box B . How many possible outcomes are there?

Ans: There are 2 tasks: select from box A (8 possibilities), or select from box B (12 possibilities). Since the sets are disjoint there are $8 + 12 = 20$ possibilities.

(b) Suppose the instructions are to select **two** toys, one from box A **and** one from box B . How many possible outcomes are there?

Ans: These are the same 2 tasks, but *both* must be done. So, multiply the numbers $8 \cdot 12 = 96$.

2. The 12-letter string "PACIFICATION" has 3 occurrences of ‘ I ’ and 2 occurrences each of the letters ‘ A ’ and ‘ C ’, and no other repetitions.

(a) How many different 12-letter strings are arrangements of this string?

Ans: $\frac{12!}{3!2!2!}$.

(b) How many arrangements of this string contain the three substrings "III", "AA" and "CC"?

Ans: Permutations of the 8 objects, ‘III’, ‘AA’, ‘CC’, ‘P’, ‘F’, ‘T’, ‘O’, and ‘N’: $8!$ arrangements.