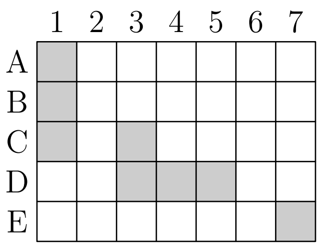
Math 3103 Combinatorics (Luecking)

Practice Quiz

1. This table represents possible seating of 5 people among 7 seats. A shaded square represents a forbidden seat assignment. The rook polynomial of the shaded squares is

   $$1 + 8x + 20x^2 + 17x^3 + 4x^4 .$$

   Using this information, how many allowable ways are there to seat all five people? (Your final answer must use only numbers and the operations of addition, subtraction, multiplication, division, powers and factorials.)



2. In the following equation, n is a nonnegative integer; the variables y_1, y_2 and y_3 are integers that must satisfy the given conditions.

   $$y_1 + y_2 + y_3 = n$$
   $$10 \leq y_1 \leq 24$$
   $$15 \leq y_2$$
   $$0 \leq y_3 \leq 14$$

   (a) Write down the generating function for the number of solutions. Write it as a quotient in which the denominator is a power of $(1 - x)$ and the numerator is a short polynomial expanded (mutiplied out) to a sum of powers of x.

   (b) Using the generating function you found in part (a), find the number of solutions when $n = 100$. You may use the notations $C(n, k)$ or $\binom{n}{k}$ in your solution.

1. $P(7,5) - 8P(6,4) + 20P(5,3) - 17P(4,2) + 4P(3,1) - 0P(2,0)$
$= \frac{7!}{2!} - 8\frac{6!}{2!} + 20\frac{5!}{2!} - 17\frac{4!}{2!} + 4\frac{3!}{2!}$

2. (a) For y_1: $x^{10} + x^{11} + \cdots + x^{24} = \frac{x^{10} - x^{25}}{1-x}$.

   For y_2: $x^{15} + x^{21} + x^{22} + \cdots = \frac{x^{15}}{1-x}$

   for y_3: $1 + x + x^2 + \cdots x^{14} = \frac{1-x^{15}}{1-x}$

   The above functions multiplied:

   $\frac{x^{10} - x^{25}}{1-x} \cdot \frac{x^{15}}{1-x} \cdot \frac{1-x^{15}}{1-x} = \frac{x^{25} - 2x^{40} + x^{55}}{(1-x)^3}$

   (b) The generating function can be rewritten as $(x^{25} - 2x^{40} + x^{55}) \sum_{j=0}^{\infty} \binom{j+2}{j} x^j$

   The terms that produce x^{100} are: $\binom{77}{75} x^{25} x^{75} - 2\binom{62}{60} x^{40} x^{60} + \binom{47}{45} x^{55} x^{45}$.

   Number of solutions: $\binom{77}{75} - 2\binom{62}{60} + \binom{47}{45}$