Matrices

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We can address the elements (or entries) in a matrix by specifying the row and the column of that entry (always in that order). If we say the ij-entry, we mean the entry in row i and column j. For example, an $n \times k$ matrix with unspecified entries would look like

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1k} \\ a_{21} & a_{22} & \cdots & a_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nk} \end{pmatrix}$$

Thus, the ij-entry is a_{ij} and this is sometimes abbreviated $A = (a_{ij})$.

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$$\begin{aligned} 2x_1 - x_2 &= 3 \\ 3x_1 + 2x_2 &= 1 \end{aligned}$$

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The above illustrates a convention our book uses: If a matrix is specified, the corresponding bold lower case letter with a subscript represents the corresponding column of the matrix. If there is an arrow over the letter, that represents the corresponding row. Finally, we can represent a matrix as a list of column vectors or a stack of row vectors as follows

$$A = (\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_k), \text{ or } \begin{pmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \cdots & \mathbf{a}_k \end{pmatrix}, \quad B = \begin{pmatrix} \vec{\mathbf{b}}_1 \\ \vec{\mathbf{b}}_2 \\ \vdots \\ \vec{\mathbf{b}}_n \end{pmatrix}$$

Arithmetic with matrices

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$$\text{Example:} \quad \begin{pmatrix} 2 & 4 \\ -1 & 6 \end{pmatrix} + \begin{pmatrix} -3 & 1 \\ 5 & 2 \end{pmatrix} = \begin{pmatrix} -1 & 5 \\ 4 & 8 \end{pmatrix}$$

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Note: you cannot add matrices that are differnt sizes. If A is $n \times k$ then you can add $A + B$ only if B is also $n \times k$

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Note that the result is a number, or it can be viewed as a 1×1 matrix.

Next case: an $n \times k$ matrix times a $k \times 1$ column vector. In this case each row of the first matrix is multiplied by the column vector.

So: $$A\mathbf{b} = \begin{pmatrix} \vec{\mathbf{a}}_1 \\ \vec{\mathbf{a}}_2 \\ \vdots \\ \vec{\mathbf{a}}_n \end{pmatrix} \mathbf{b} = \begin{pmatrix} \vec{\mathbf{a}}_1\mathbf{b} \\ \vec{\mathbf{a}}_2\mathbf{b} \\ \vdots \\ \vec{\mathbf{a}}_n\mathbf{b} \end{pmatrix}$$

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For example: $$\begin{pmatrix} 2 & 3 & -1 \\ 1 & -2 & 3 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 6+6-1 \\ 3-4+3 \end{pmatrix} = \begin{pmatrix} 11 \\ 2 \end{pmatrix}$$

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If A is $n \times k$ and B is $k \times m$ then: $AB = \begin{pmatrix} A\mathbf{b}_1 & A\mathbf{b}_2 & \cdots & A\mathbf{b}_m \end{pmatrix}$

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\begin{pmatrix} 2 & 3 \\ 1 & 2 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 2 \end{pmatrix}
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If A is an $n \times k$ matrix and B is $k \times m$ then we can multiply AB and the result is $n \times m$. We can multiply BA only if $m = n$, the result may be an entirely different size from AB. Even if they are the same size, AB and BA are typically not equal.

Some special cases.

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Then the equation $A\mathbf{x} = \mathbf{b}$ is the same as

$$A\mathbf{x} = \begin{pmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1k}x_k \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2k}x_k \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nk}x_k \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} = \mathbf{b}$$

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Equating the positions in $A\mathbf{x}$ to the corresponding positions in $\mathbf{b}$ gives us a system of equations.

Another special case is confusing at first: a column vector times a row vector. This can always be done: as it is an $n \times 1$ times a $1 \times k$. The rows of the first have length 1 as do the columns of the second. The result will be $n \times k$:

$$\mathbf{a}\vec{\mathbf{b}} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} \begin{pmatrix} b_1 & b_2 & \cdots & b_k \end{pmatrix} = \begin{pmatrix} a_1b_1 & a_1b_2 & \cdots & a_1b_k \\ a_2b_1 & a_2b_2 & \cdots & a_2b_k \\ \vdots & \vdots & \ddots & \vdots \\ a_nb_1 & a_nb_2 & \cdots & a_nb_k \end{pmatrix}$$

This is sometimes called the *outer product* of $\mathbf{a}$ and $\vec{\mathbf{b}}$.